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To Bet or Not To Bet: The Modified Kelly Criterion

Dani Chu, Yifan Wu & Tim B. Swartz

Sports Gambling

- **Point Spread:**

Golden State Warriors	+ 6.5	1.909
Sacramento Kings	- 6.5	1.909

- **Over/Under:**

Capitals vs Penguins	Over	5.5 goals	1.90
Capitals vs Penguins	Under	5.5 goals	1.90

- **Other Props:**

Belichick Hoodie Colour	Blue	1.92
Belichick Hoodie Colour	Grey	1.92

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Profitable Systems

- A gambling system (often found in sports) is profitable with
  - Wager of size $x
  - System win probability *p*
  - Return of $x • *θ* on a win and *0* on a loss

If

$$(-x)(1-p) + (x\theta - x)p > 0 \rightarrow p > 1/\theta$$

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The Kelly Criterion

- The Kelly criterion (Kelly 1956) provides a gambler an optimal fraction of a bankroll for wagering given probability p of winning a bet.

$$k(p) = \begin{cases} \frac{p\theta - 1}{\theta - 1} & p > 1/\theta \\ 0 & p \leq 1/\theta \end{cases}$$

- **Problem:** Experienced gamblers claim $k(p)$ is too large
- **Reason:** p is not known and often overestimated with data
- **The Fix:** Model the unknown parameter p and estimate the unknown $k(p)$ with the estimator $f = f(x)$

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Modified Kelly Criterion

- To assess the quality of f we use $l_i(f, p)$ as loss function i
- Use a Bayes estimator f which minimizes the Bayes risk
  - Will minimize expected posterior loss

$$G(f) = \int_0^1 l_0(f, p)\pi(p \mid x)dp$$

Modified Kelly Criterion

- Posterior distribution of p is defined by
  - historical data $x \sim \text{Binomial}(n, p)$ from historical win/loss data
  - prior distribution $p \mid \sim \text{Beta}(a, b)$
- For different loss functions (see poster) we get different f
  - Can be solved for directly or by estimating the integral through computation

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In Practice

- **Find a Winning Gambling System**
  - Professor MJ's push-back phenomenon in NBA playoff games point spread system with 271/484 = 0.56 win rate.

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  - $a = b = 50$
    - Centered at 0.5, and ~ 95% of the prior probability in the interval (0.4, 0.6)

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  - $l_{3b}(f, p) = (I_{f > k(p)} + 2)\ |f - k(p)|^{1.5}$ - lies halfway between absolute error loss and squared error loss. Penalty of overestimation is 1.5 times worse than underestimation

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- Starting with an $1000 bankroll would have produced a final bankroll of
  - Original Kelly: $668.34, using 7.6% of initial bankroll for each bet
  - Modified Kelly: $794.89, using 4.7% of initial bankroll for each bet

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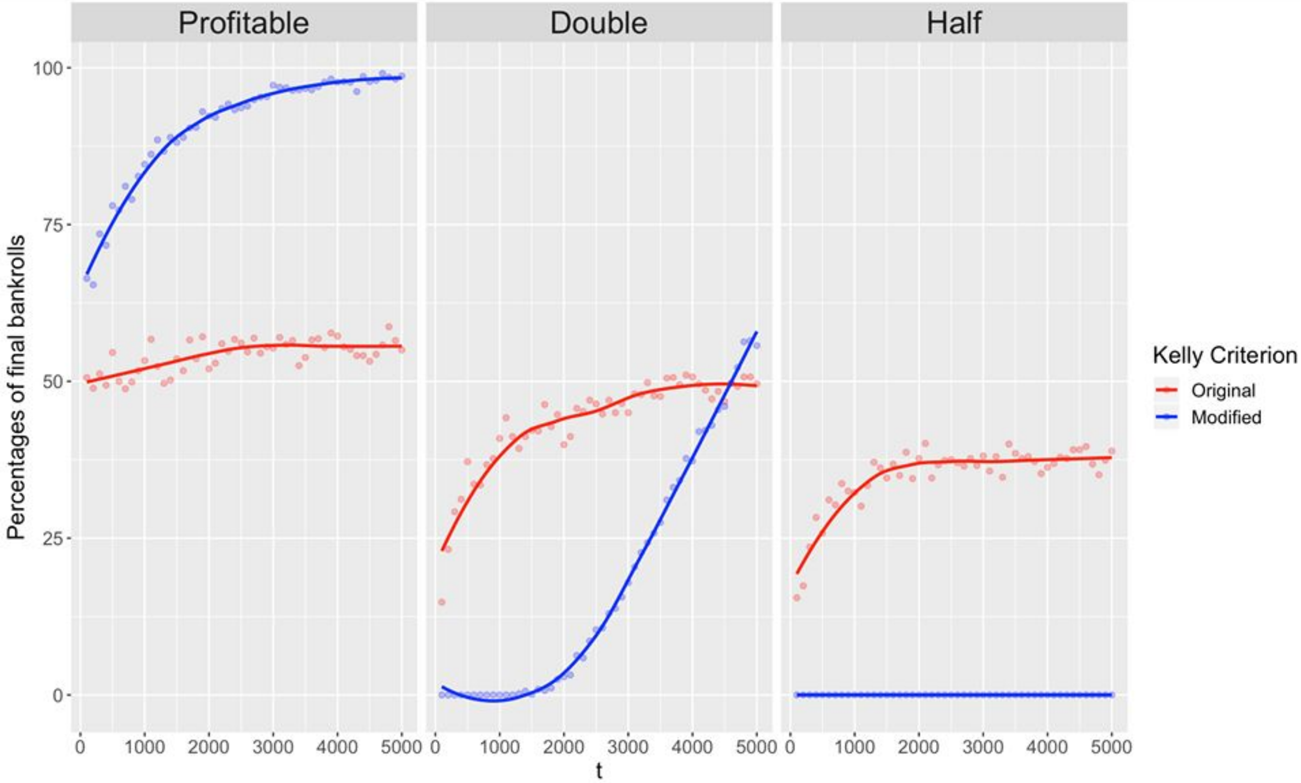
In Simulation



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THANKS!

Any Questions?

Please find me at the E-Poster Session, Poster 11!
Or see our paper in JQAS!

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Are you a student interested in Sports Analytics?
Enjoy Vancouver and want to come back on September 22nd?
Check out the Vancouver Whitecaps Datathon at www.VanSASH.com!

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